

A No-Scale Supergravity Realization of the Starobinsky Model

John Ellis^{a,*}, Dimitri V. Nanopoulos^{b,†} and Keith A. Olive^{c,‡}

^a *Theoretical Particle Physics and Cosmology Group, Department of Physics,
King's College London, London WC2R 2LS, United Kingdom;
Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

^b *George P. and Cynthia W. Mitchell Institute for Fundamental Physics and Astronomy,
Texas A&M University, College Station, TX 77843, USA;
Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA;
Academy of Athens, Division of Natural Sciences,
28 Panepistimiou Avenue, Athens 10679, Greece, and
^c *William I. Fine Theoretical Physics Institute, School of Physics and Astronomy,
University of Minnesota, Minneapolis, MN 55455, USA**

We present a model for cosmological inflation based on a no-scale supergravity sector with an $SU(2,1)/U(1)$ Kähler potential, a single modulus T and an inflaton superfield Φ described by a Wess-Zumino model with superpotential parameters (μ, λ) . This model yields a scalar spectral index n_s and a tensor-to-scalar ratio r that are compatible with the Planck measurements for values of $\lambda \simeq \mu/3M_P$. For the specific choice $\lambda = \mu/3M_P$, the model is a no-scale supergravity realization of the $R + R^2$ Starobinsky model.

The initial release of cosmic microwave background (CMB) data from the Planck satellite [1] confront theorists of cosmological inflation [2, 3] with a challenge. On the one hand, the data have many important features that are predicted qualitatively by the inflationary paradigm. For example, there are no significant signs of non-Gaussian fluctuations or hints of non-trivial topological features such as cosmic strings, and the spectrum of scalar density perturbations exhibits a significant tilt: $n_s \simeq 0.960 \pm 0.007$, as would be expected if the effective scalar energy density decreased gradually during inflation. On the other hand, many previously popular field-theoretical models of inflation are ruled out by a combination of the constraint on n_s and the tensor-to-scalar ratio $r < 0.08$ as now imposed by Planck *et al*: see, e.g., Fig. 1 of [1]. The only model with truly successful predictions displayed in Fig. 1 of [1] is the R^2 inflation model of Starobinsky [4].

Since the field energy during the inflationary epoch is typically $\ll M_P^4$, it is natural to study renormalizable models, i.e., some combination of ϕ^2 , ϕ^3 and ϕ^4 in the single-field case. In this spirit, it was shown in [5] that a single-field model with a potential of the form

$$V = A\phi^2(v - \phi)^2 \quad (1)$$

could easily produce Planck-compatible values of (n_s, r) for a suitable number of e-folds before the end of inflation $N \sim 50$ to 60. This simple symmetry breaking potential has a long pedigree, having been proposed initially in [6] (for a review, see [2]) where it was argued that sufficient inflation required $v > M_P$.

We think that a natural framework for constructing inflationary models is supersymmetry. As we pointed out in [7], in addition to all the well-known reasons for postulating low-scale supersymmetry, the small values of the quartic and quadratic couplings that would be required in a successful inflationary model become technically natural in the presence of low-scale supersymmetry. Small values of $\delta\rho/\rho$ become technically natural if low-scale supersymmetry is invoked [7], and if the GUT Higgs is distinguished from the singlet field that produces inflation, that later became known as the inflaton [8].

The simplest globally-supersymmetric model is the Wess-Zumino model with a single chiral superfield Φ [9], which is characterized by a mass term $\hat{\mu}$ and a trilinear coupling λ , with the superpotential

$$W = \frac{\hat{\mu}}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3. \quad (2)$$

As was discussed in [5], the effective potential of the Wess-Zumino model reduces to (1) when the imaginary part of the scalar component of Φ vanishes, in which case this model yields Planck-compatible inflation.

However, global symmetry is not enough. In the context of early-Universe cosmology one should certainly include gravity and hence construct a locally-supersymmetric model, i.e., upgrade to supergravity [10]. The first attempt at constructing an inflationary model in $N = 1$ supergravity proposed a generic form for the superpotential for a single inflaton [11], the simplest form being $W = m^2(1 - a\Phi)^2$ [12]. As discussed in [13], while this relatively simple model is capable of sufficient inflation, it is an example of accidental inflation in the sense that the coefficient of the linear term in the superpotential, a , must be extremely close to unity. This model has also become one of Planck's casualties. The scalar-to-tensor ratio in this model is very small, but the value of

* John.Ellis@cern.ch

† dimitri@physics.tamu.edu

‡ olive@physics.umn.edu

n_s predicted in this model is $n_s \simeq 1 - 4/N = 0.933$ for $N = 60$ [14].

In a supergravity model with a generic Kähler potential for the chiral supermultiplets there are quadratic $|\phi|^2$ terms in the effective potential that destroy its suitability for inflation, an obstacle known as the η -problem [3]. As was pointed out in [15], a natural solution to this problem is offered by no-scale supergravity [16], in which there is a non-compact $SU(N,1)/U(1)$ symmetry, such quadratic terms are suppressed, and the effective scalar potential resembles that in a globally-supersymmetric model. Other no-scale supergravity approaches have also been proposed [17], as well as models based on a non-compact Heisenberg symmetry [18], a shift symmetry [19–21], or string theory [22]. The $SU(N,1)$ model [15] was based on the superpotential $W = m^2(\phi - \phi^4/4)$ and gives similar predictions for the inflationary parameters as the minimal $N = 1$ model discussed above. This too is an example of accidental inflation [13] and a small change in the coefficient of the quartic term would lead to parameters consistent with Planck data [1].

In this paper we show how one can elevate the simplest globally-supersymmetric Wess-Zumino inflationary model of [5] to a no-scale supergravity version (NSWZ). Concretely, we study a model in which the inflaton superfield is embedded in an $SU(2,1)/U(1)$ no-scale supergravity sector together with a modulus field T and find a range of the parameters where it is compatible with the Planck data [1]. Quite remarkably, as we show, the NSWZ model is the conformal equivalent of an $R + R^2$ model of gravity for one specific value of $\hat{\mu}/\lambda$, so that in this case our realization of inflation in the NSWZ model is equivalent to the Starobinsky model of inflation [4].

We first recall the basic relevant formulae governing the kinetic term and the effective potential of scalar fields ϕ in $\mathcal{N} = 1$ supergravity, specializing to the no-scale case with non-compact $SU(N, 1)/U(1)$ symmetry. The scalar sector may be characterized in general by a hermitian Kähler function K and a holomorphic superpotential W via the combination $G \equiv K + \ln W + \ln W^*$. The kinetic term is then given by $K_i^{j*} \partial_\mu \phi^i \partial_\mu \phi_j^*$, where the Kähler metric $K_i^{j*} \equiv \partial^2 K / \partial \phi^i \partial \phi_j^*$, and the effective potential is

$$V = e^G \left[\frac{\partial G}{\partial \phi^i} K_{j*}^i \frac{\partial G}{\partial \phi_j^*} - 3 \right], \quad (3)$$

where K_{j*}^i is the inverse of the Kähler metric K_i^{j*} .

In the minimal no-scale $SU(2, 1)/U(1)$ case, there are two complex scalar fields: T , a modulus field, and ϕ , which we identify as the inflaton field, with the Kähler function $K = -3 \ln(T + T^* - |\phi|^2/3)$. In this case, the kinetic terms for the scalar fields T and ϕ become

$$\mathcal{L}_{KE} = (\partial_\mu \phi^*, \partial_\mu T^*) \left(\frac{3}{(T + T^* - |\phi|^2/3)^2} \right)$$

$$\begin{pmatrix} (T + T^*)/3 & -\phi \\ -\phi^* & 1 \end{pmatrix} \begin{pmatrix} \partial^\mu \phi \\ \partial^\mu T \end{pmatrix}, \quad (4)$$

and the effective potential becomes

$$V = \frac{\hat{V}}{(T + T^* - |\phi|^2/3)^2} : \hat{V} \equiv \left| \frac{\partial W}{\partial \phi} \right|^2. \quad (5)$$

In early no-scale models [15, 18] it was assumed that K was fixed so that the potential up to a re-scaling was simply \hat{V} . Here we assume that the T field has a vacuum expectation value (vev) $2\langle \text{Re} T \rangle = c$ and $\langle \text{Im} T \rangle = 0$ that is determined by non-perturbative high-scale dynamics, as in the KKL T [23] or KL models [24, 25]. In this case, we may neglect the kinetic mixing between the T and ϕ fields in (4), and are left with the following effective Lagrangian for the inflaton field ϕ :

$$\mathcal{L}_{eff} = \frac{c}{(c - |\phi|^2/3)^2} |\partial_\mu \phi|^2 - \frac{\hat{V}}{(c - |\phi|^2/3)^2}. \quad (6)$$

We assume as in [5] the minimal Wess-Zumino superpotential (2) for the inflaton field.

To better study the potential for the inflaton, we first transform ϕ to the field χ :

$$\phi = \sqrt{3c} \tanh \left(\frac{\chi}{\sqrt{3}} \right). \quad (7)$$

With this field redefinition, the Lagrangian becomes

$$\mathcal{L}_{eff} = \text{sech}^2((\chi - \chi^*)/\sqrt{3}) \left[|\partial_\mu \chi|^2 - \left(\frac{3}{c} \right) \left| \sinh(\chi/\sqrt{3}) \left(\hat{\mu} \cosh(\chi/\sqrt{3}) - \sqrt{3c}\lambda \sinh(\chi/\sqrt{3}) \right) \right|^2 \right]. \quad (8)$$

Clearly the vev of the T field can be absorbed into the definition of the mass and, writing $\hat{\mu} = \mu\sqrt{c}/3$, the potential becomes

$$V = \mu^2 \left| \sinh(\chi/\sqrt{3}) \left(\cosh(\chi/\sqrt{3}) - \frac{3\lambda}{\mu} \sinh(\chi/\sqrt{3}) \right) \right|^2. \quad (9)$$

Writing χ in terms of its real and imaginary parts: $\chi = (x + iy)/\sqrt{2}$ and, for reasons which will become clear, considering the specific case where the quartic coupling $\lambda = \mu/3$ (in Planck units), so that we have

$$\mathcal{L}_{eff} = \frac{1}{2} \sec^2(\sqrt{2/3}y) ((\partial_\mu x)^2 + (\partial_\mu y)^2) - \mu^2 \frac{e^{-\sqrt{2/3}x}}{2} \sec^2(\sqrt{2/3}y) \left(\cosh \sqrt{2/3}x - \cos \sqrt{2/3}y \right)^2. \quad (10)$$

The imaginary part of the inflaton is fixed to $y = 0$ by the potential, having a mass $m_y = \mu/\sqrt{3}$ during inflation when x is large and $m_y = \mu/\sqrt{6}$ at the end of inflation when $x = 0$. Thus we expand the Lagrangian about $y = 0$, in which case we have minimal kinetic terms for x and y , accompanied by derivative interaction terms. The

potential for the real part of the inflaton now takes the form

$$V = \mu^2 e^{-\sqrt{2/3}x} \sinh^2(x/\sqrt{6}). \quad (11)$$

This potential is depicted in Fig. 1, where we also display the potential for values of λ slightly perturbed from the nominal value of $\mu/3$.

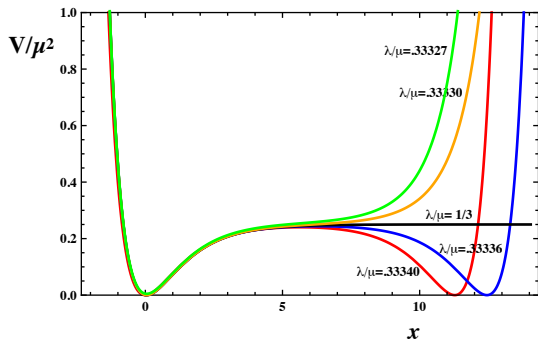


FIG. 1. The potential V in the NSWZ model for choices of $\lambda \sim \mu/3$ in Planck units, as indicated.

We use the standard slow-roll expressions for the tensor-to-scalar ratio r and the spectral index n_s for the scalar perturbations in terms of the slow-roll inflation parameters ϵ, η [3], which we evaluate in terms of the canonically-normalized field x . In the NSWZ model described above the vev of T is absorbed in the definition of the mass parameter μ , which is determined by the normalization of the quadrupole. For the special case $\lambda = \mu/3$, we have

$$A_s = \frac{V}{24\pi^2\epsilon} = \frac{\mu^2}{8\pi^2} \sinh^4(x/\sqrt{6}), \quad (12)$$

implying a value $\mu = 2.2 \times 10^{-5}$ in Planck units for $N = 55$: μ varies between $1.8 - 3.4 \times 10^{-5}$ over the range of NSWZ models considered here. Setting the remaining NSWZ parameter $\lambda = \mu/3$, we have

$$\epsilon = \frac{1}{3} \text{csch}^2(x/\sqrt{6}) e^{-\sqrt{2/3}x}, \quad (13)$$

$$\eta = \frac{1}{3} \text{csch}^2(x/\sqrt{6}) (2e^{-\sqrt{2/3}x} - 1), \quad (14)$$

which allows us to determine the quantities (n_s, r) , once the value of the field x is fixed by requiring $N = 50 - 60$ e-folds. The nominal choice of $N = 55$ yields $x = 5.35$, $n_s = 0.965$, and $r = .0035$.

Fig. 2 displays the predictions for (n_s, r) of the NSWZ model for five choices of the coupling λ that yield $n_s \in [0.93, 1.00]$ and $N \in [50, 60]$. The last 50-60 e-folds of inflation arise as x rolls to zero from $\sim 5.1 - 5.8$, the exact value depending on λ and N . As one can see, the values of λ are constrained to be close to the critical value $\mu/3$, for which we find extremely good agreement with the Planck determination of n_s . The values of r are

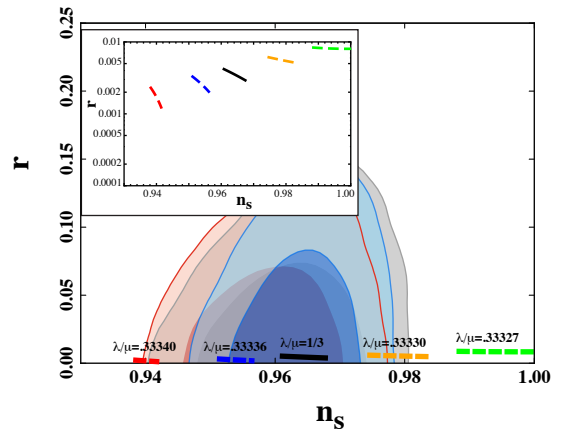


FIG. 2. Predictions from the NSWZ model for the tilt n_s in the spectral index of scalar perturbations and for the tensor-to-scalar ratio r , compared with the 68 and 95% CL regions found in analyses of Planck and other data [1]. In the main panel the lines are labelled by the values of λ/μ (in Planck units) assumed in each case. In the inset, the same cases are shown on a log scale to display better the values of r .

rather small for $\lambda = \mu/3$, varying over the range $0.0012 - 0.0084$, in the models considered.

At first sight, this success might appear to be another example of accidental inflation [13] but, as we now show, this choice of λ has a more profound geometric interpretation. The alert reader may have noticed resemblances of both the potential shown in Fig. 1 and the values of (n_s, r) found for the $\lambda = \mu/3$ model with results for inflation in the $R + R^2$ model proposed by Starobinsky [4]. To probe further this resemblance, we examine the generalization of the Einstein-Hilbert action to contain an R^2 contribution, where R is the scalar curvature,

$$S = \frac{1}{2} \int d^4x \sqrt{-g} (R + R^2/6M^2), \quad (15)$$

where $M \ll M_P$ is some mass scale. This theory is conformally equivalent to canonical gravity plus a scalar field φ [26]. Making the transformation $\tilde{g}_{\mu\nu} = (1 + \varphi/3M^2)g_{\mu\nu}$ and the field redefinition $\varphi' = \sqrt{\frac{3}{2}} \ln(1 + \frac{\varphi}{3M^2})$, we obtain the action

$$S = \frac{1}{2} \int d^4x \sqrt{-\tilde{g}} \left[\tilde{R} + (\partial_\mu \varphi')^2 - \frac{3}{2} M^2 (1 - e^{-\sqrt{2/3}\varphi'})^2 \right], \quad (16)$$

corresponding to a potential

$$V = \frac{3}{4} M^2 (1 - e^{-\sqrt{2/3}\varphi'})^2. \quad (17)$$

The potential (17) is identical with the potential (11) along the real direction of the NSWZ model! Moreover, we have the identification $M^2 = \mu^2/3$, which is $\hat{\mu}^2$ for $c = \langle (T + T^*) \rangle = 1$. Thus the Starobinsky mass M is directly related to the NSWZ mass $\hat{\mu}$ in the superpotential (2).

We have shown in this paper that the simplest $SU(2,1)/U(1)$ no-scale supergravity model with a single

modulus field T and a single matter field ϕ with the simplest renormalizable Wess-Zumino superpotential, identified with the inflaton, is capable of yielding cosmological inflation with values of the scalar spectral tilt n_s and the tensor-to-scalar ratio r within the region favoured by Planck and other data at the 68% CL. Successful inflation is obtained for $\lambda \simeq \mu/3$ in Planck units. This NSWZ model is a proof of the existence of acceptable models of inflation based on no-scale supergravity, and normally we would not advocate that its details should necessarily be taken literally. For example, a realistic no-scale model derived from a generic compactification of string theory would have more moduli fields, with many matter fields that could be the inflaton, with a superpotential more complicated than assumed here.

However, it is truly striking that the NSWZ model is conformally equivalent to the Starobinsky R^2 model [4] for the specific choice $\lambda = \mu/3$ in Planck units. This cor-

respondence suggests that there is a profound geometric interpretation of this model that remains to be understood.

Acknowledgements. J.E. thanks Djuna Croon, Mirjam Cvetič, Nick Mavromatos, Henry Tye and Gary Shiu for discussions, and K.A.O. thanks Andrei Linde and Misha Voloshin for discussions. The work of J.E. was supported in part by the London Centre for Terauniverse Studies (LCTS), using funding from the European Research Council via the Advanced Investigator Grant 267352. The work of D.V.N. was supported in part by the DOE grant DE-FG03-95-Er-40917. The work of K.A.O. was supported in part by DOE grant DE-FG02-94ER-40823 at the University of Minnesota.

-
- [1] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].
 - [2] K. A. Olive, Phys. Rept. **190** (1990) 307.
 - [3] See, for example: A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990); D. H. Lyth and A. Riotto, *Phys. Rep.* **314** (1999) 1 [arXiv:hep-ph/9807278]. J. Martin, C. Ringeval and V. Vennin, arXiv:1303.3787 [astro-ph.CO].
 - [4] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980); V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)]; A. A. Starobinsky, Sov. Astron. Lett. **9**, 302 (1983).
 - [5] D. Croon, J. Ellis and N. E. Mavromatos, arXiv:1303.6253 [astro-ph.CO].
 - [6] A. Albrecht and R. H. Brandenberger, Phys. Rev. D **31**, 1225 (1985).
 - [7] J. R. Ellis, D. V. Nanopoulos, K. A. Olive and K. Tamvakis, Phys. Lett. B **118** (1982) 335; Phys. Lett. B **120** (1983) 331; Nucl. Phys. B **221** (1983) 52.
 - [8] D. V. Nanopoulos, K. A. Olive and M. Srednicki, Phys. Lett. B **127**, 30 (1983).
 - [9] J. Wess and B. Zumino, Nucl. Phys. B **70** (1974) 39.
 - [10] D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, Phys. Rev. D **13** (1976) 3214; S. Deser and B. Zumino, Phys. Lett. B **62** (1976) 335.
 - [11] D. V. Nanopoulos, K. A. Olive, M. Srednicki and K. Tamvakis, Phys. Lett. B **123**, 41 (1983).
 - [12] R. Holman, P. Ramond and G. G. Ross, Phys. Lett. B **137**, 343 (1984).
 - [13] A. D. Linde and A. Westphal, JCAP **0803**, 005 (2008) [arXiv:0712.1610 [hep-th]].
 - [14] This model still falls within the region allowed by Planck at the 95% CL for $N = 70$ or a slight deviation in a from unity, by 1 part in 10^6 .
 - [15] J. R. Ellis, K. Enqvist, D. V. Nanopoulos, K. A. Olive and M. Srednicki, Phys. Lett. B **152** (1985) 175 [Erratum-ibid. **156B** (1985) 452].
 - [16] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B **133** (1983) 61; J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B **247** (1984) 373.
 - [17] A. S. Goncharov and A. D. Linde, Class. Quant. Grav. **1**, L75 (1984).
 - [18] P. Binetruy and M. K. Gaillard, Phys. Lett. B **195** (1987) 382; H. Murayama, H. Suzuki, T. Yanagida and J. Yokoyama, Phys. Rev. D **50**, 2356 (1994) [arXiv:hep-ph/9311326]; S. Antusch, M. Bastero-Gil, K. Dutta, S. F. King and P. M. Kostka, Phys. Lett. B **679** (2009) 428 [arXiv:0905.0905 [hep-th]].
 - [19] M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. Lett. **85** (2000) 3572 [hep-ph/0004243]; K. Nakayama, F. Takahashi and T. T. Yanagida, arXiv:1303.7315 [hep-ph].
 - [20] S. C. Davis and M. Postma, JCAP **0803**, 015 (2008) [arXiv:0801.4696 [hep-ph]].
 - [21] R. Kallosh and A. Linde, JCAP **1011**, 011 (2010) [arXiv:1008.3375 [hep-th]]; R. Kallosh, A. Linde and T. Rube, Phys. Rev. D **83**, 043507 (2011) [arXiv:1011.5945 [hep-th]]; R. Kallosh, A. Linde, K. A. Olive and T. Rube, Phys. Rev. D **84**, 083519 (2011) [arXiv:1106.6025 [hep-th]].
 - [22] E. Silverstein and A. Westphal, Phys. Rev. D **78**, 106003 (2008) [arXiv:0803.3085 [hep-th]]; L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D **82**, 046003 (2010) [arXiv:0808.0706 [hep-th]]. X. Dong, B. Horn, E. Silverstein and A. Westphal, arXiv:1011.4521 [hep-th].
 - [23] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240].
 - [24] R. Kallosh and A. D. Linde, JHEP **0412**, 004 (2004) [arXiv:hep-th/0411011]; J. J. Blanco-Pillado, R. Kallosh and A. D. Linde, JHEP **0605**, 053 (2006) [hep-th/0511042]; R. Kallosh and A. D. Linde, JCAP **0704**, 017 (2007) [arXiv:0704.0647 [hep-th]].
 - [25] For a recent discussion of moduli stabilization, see M. Cicoli, S. de Alwis and A. Westphal, arXiv:1304.1809 [hep-th] and references therein.
 - [26] B. Whitt, Phys. Lett. B **145**, 176 (1984).